Methodological Background

Curvelet Transform

Curvelet space is the core of Hybrid Adaptive Filtering (HAF) section of the proposed scheme that aims to spotlight the structural and textural characteristics of Crohn's Disease (CD) lesions and facilitate feature extraction. Among the most popular methods for characterizing the textural appearance of surfaces are wavelets and curvelets that offer multi-resolution analysis. The rationale for engaging multi-resolution analysis in Wireless Capsule Endoscopy (WCE) images is that CD lesions are characterized by great variations in appearance in terms of scale, shape, size, illumination and orientation. Additionally, the images contain a significant amount of background variation. Consequently, a robust tool is needed that is able to capture structural/textural data in various scales and directions.

Wavelets have been commonly used for multi-resolution two-dimensional (2D) signal analysis. The power of wavelet transform (WT) rests on its ability to successfully capture point singularities, for piecewise smooth functions in one dimension (1D). Unfortunately, this is not the case in two dimensions. In essence, 2D piecewise smooth signals, such as images, exhibit 1D singularities (edges) that cannot be efficiently described by wavelets. That is, edges separate smooth regions and while they are discontinuous across, they are typically smooth curves. In the 2D case, wavelets are produced by a tensor product of 1D wavelets and, thus, are good at describing discontinuities at edge points, but cannot capture the smoothness along edges. In other words, WT isolates directional data that only capture horizontal, vertical and diagonal structures in an image. Such a directional selectivity is not sufficient to describe medical images.

In an attempt to overcome this traditional weakness of WT, Candes *et al.* introduced curvelet transform (CT)[1]. Its key concept is to represent a curve as a superposition of functions of various lengths and widths obeying a specific scaling law[2]. Continuous CT is defined by a radial window $W(r)$ and an angular window $V(\theta)$ that are both smooth, nonnegative and real-valued. Considering U_j as the Fourier transform of a function $\varphi_j(x)$, we can assume $\varphi_j(x)$ as a "mother" curvelet in the sense that all curvelets at scale 2^{-j}, orientation θ_l and position $x_k(j,l)$ are obtained by rotations, scaling and translations of φ_j. A curvelet coefficient is then defined as the inner product between an element $f \epsilon L^2 (R^2)$ and a curvelet $\varphi_{j,l,k}(x) = \varphi_j(R_{\theta_l}(x - x_k^{(j,l)}))$ at scale 2^{-j}, orientation θ_l, and position $x_k(j,l) = R_{\theta_l}^{-1}(k_1 \cdot 2^{-j}, k_2 \cdot 2^{-j/2})$, where R_θ is the rotation matrix by θ radians. The needle shaped elements of CT exhibit high directional sensitivity; hence, depicting more efficiently singularities along curves than traditional WT and providing better texture discrimination ability than wavelet counterparts[2]. The continuous CT can be extended to the digital space via either unequispaced Fast Fourier Transform (FFT) or wrapping. Both techniques have the same complexity, however, the wrapping algorithm is somewhat simpler and, thus, more popular[2].

Differential Lacunarity (DLac) Analysis

DLac is a robust tool for multi-scale and translation invariant texture analysis, capable to reveal slight or sharp changes in neighboring pixels without directional limitation, necessary in the case of WCE data. DLac has been used in various pattern discrimination problems in various scientific fields[3-5].

Lacunarity

Lacunarity (Lac), derived from the word lacuna meaning "gap", was introduced by Mandelbrot[6] as a means to discriminate textures and natural surfaces that share the same fractal dimension, but significantly vary in visual appearance. Fractal dimension does not fully describe the space-filling characteristics of data, since it measures how much space is filled. To this end, Lac is a counterpart to fractal dimension that describes the texture of a fractal by measuring how data fill space. So, Lac has been used as a more general technique to characterize patterns of spatial dispersion[7]. More specifically, Lac analysis evaluates the largeness and distribution of gaps or holes in data sets at multiple scales. The more gaps distributed across a broad range of sizes a set contains, the higher Lac value it exhibits. Beyond being an intuitive measure of "gappiness", Lac analysis can quantify additional features of patterns, such as translational and rotational invariance and, more generally, heterogeneity. Gefen *et al.*[8] defined Lac as the deviation of a fractal from translational invariance. Sets with non-uniform distribution of gaps can be considered heterogeneous and exhibit higher Lac than almost transnationally invariant (homogeneous) sets. But, translational invariance is highly scale-dependent. Sets that are homogeneous at small scales can be quite heterogeneous when examined at larger scales and vice versa. Lac, can deal with this situation due to its inherent characteristics. From this perspective, Lac analysis can be considered as a scale-dependent measure of texture of an object[6,7]. A number of methods have been presented to calculate Lac, but the most popular algorithms are founded on the intuitively clear and simple Gliding Box Algorithm (GBA)[9]. GBA is functional on 1-D binary data; Plotnick *et al.*[7], however, extended the concept of Lac to real datasets by applying thresholding.

Differential Lacunarity

In order to process grayscale images with Lac, the most straightforward approach is to extend the original GBA algorithm to 2D and convert the grayscale images to binary through thresholding. Nonetheless, in many scientific fields, and especially in medical imaging, such a thresholding procedure discards valuable information and cannot always be performed. To address this shortcoming, Dong[10] proposed a new version of Lac, i.e., DLac, appropriate for grayscale image analysis. The calculation of DLac is based on a "Differential Box Counting" method that utilizes a gliding box R ($r \cdot r$ pixels) and a gliding window W ($w \cdot w$ pixels, with $r < w$). W scans the image, while R scans W. Both W and R move in an overlapping pattern, sliding one pixel at a time. R is used for the calculation of the "box mass" M of the window at every position. If $Q(M, r)$ is the probability function of M distribution across the image, the DLac of the image at scale r, w is defined as[10]

$$\Lambda(w,r) = \sum_M M^2 Q(M,w,r,) / [\sum_M MQ(M,w,r,)]^2. \quad (1)$$

It is common practice to calculate DLac for a variety of scales, forming a DLac–w curve. This curve is the multi-scale description of texture and characterises the specific space-filling pattern.

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Nomenclature

Abbreviation		Definition
$\overline{ACC}$	…	Average accuracy
AR	…	Autoregressive model
CaEn	…	CapsuleEndoscopy.org database
CD	…	Crohn’s Disease
CT	…	Curvelet transform
CurvLac	…	Methodology based on CT and Lac[6]
CurvLBP	…	Methodology based on CT and LBP[8]
DLac	…	Differential lacunarity
ECT	…	Methodology based on edge, color and texture features[17]
EFF	…	Energy-based fitness function
FICE	…	Fuji Intelligent Chromo Endoscopy
FF	…	Fitness function
FFT	…	Fast Fourier transform
FV	…	Feature vector
GA	…	Genetic algorithm
GBA	…	Gliding box algorithm
GI	…	Gastrointestinal
Grad	…	Gradient-based features
GT	…	Gastrointestinal tract
HAF	…	Hybrid adaptive filtering
Hist	…	Histogram-based features
HSV	…	Hue-Saturation-Value
IP	…	Initial population
Lac	…	Lacunarity

LBP	…	Local binary patterns
LFF	…	Lacunarity curve gradient fitness function
NR	…	No reconstruction
$\overline{PREC}$	…	Average precision
R	…	Reconstruction
RGB	…	Red-Green-Blue
riuLBP	…	Rotation invariant uniform local binary patterns
RLM	…	Run-length-matrix
ROI	…	Region of interest
SB	…	Small bowel
$\overline{SENS}$	…	Average sensitivity
SIFT	…	Scale invariant feature transform
$\overline{SPEC}$	…	Average specificity
SVM	…	Support vector machines
WCE	…	Wireless capsule endoscopy
WT	…	Wavelet transform